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Chebyshev finite difference method for the effects of variable viscosity and variable thermal conductivity on heat transfer from moving surfaces with radiation

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Abstract

In this article, we studied the effects of variable viscosity and variable thermal conductivity on heat transfer from moving surfaces in a micropolar fluid through a porous medium with radiation. The fluid viscosity is assumed to vary as an inverse linear function of temperature and the thermal conductivity is assumed to vary as a linear function of temperature. The governing fundamental equations are approximated by a system of nonlinear ordinary differential equations and are solved numerically by using Chebyshev finite difference method (ChFD). Numerical solutions are obtained for different values of variable viscosity, variable thermal conductivity, porous medium, radiation, velocity ratio and micropolar parameters. Two cases are considered, one corresponding to a plane surface moving in parallel with the free stream and the other, a surface moving in the opposite direction to the free stream. The numerical results show that, variable viscosity, variable thermal conductivity, radiation and the permeability have significant influences on the velocity, the angular velocity and temperature profiles, shear stress, couple stress and Nusselt number in the above two cases.

Keywords: Micropolar fluid; Variable viscosity; Variable thermal conductivity; Radiation; Porous medium; Moving surfaces; Heat transfer; Chebyshev finite difference

1. Introduction

Most of the existing analytical studies for this problem are based on the constant physical properties of the ambient fluid [1–3]. However, it is know that these properties may change with temperature [4]. To accurately predict the flow and heat transfer rates, it is necessary to take into account this variation of viscosity and thermal conductivity. The effect of radiation on hydrodynamic flow and heat transfer problems have become more important industrially. Many processes in engineering areas occur at high temperatures and acknowledge radiation heat transfer becomes very important for the design of pertinent equipment.

Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. At high operating temperature, radiation effect can be quite significant (see [5–7]). The porous media heat transfer problems have several practical engineering applications such as geothermal systems, crude oil extraction, ground water pollution and another many practical applications such as in biomechanical problems, e.g., blood, flow in the pulmonary alveolar sheet and in filtration transpiration cooling. The study of the boundary layer behaviour on continuous surfaces is important because the analysis of such flows finds applications in different areas such as the cooling of a metallic plate in a cooling bath, the boundary layer along material handling conveyers and the boundary layer along a liquid film in condensation processes. Erigen [8] introduced the concept of a micropolar fluid in an attempt to explain the behavior of a certain fluid containing polymeric additives and naturally occurring fluids such as the phenomenon of the flow of colloidal fluids, liquid crystals, polymeric suspensions, animal blood, etc.

In studying the motion of such a fluid, the nonlinearity of the basic equation and additional mathematical difficulties associated with it has led several investigators to explore the

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Nomenclature

u, v	velocity components	$m \cdot s^{-1}$
T	temperature	K
N	angular velocity	
K	vortex viscosity	
k_f	thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
S	thermal conductivity parameter	
q_r	heat flux	$W \cdot m^{-2}$
k^*	permeability of the porous medium	m^2
c_P	specific heat at constant pressure	$kJ \cdot kg^{-1} \cdot K^{-1}$
f, g	dimensionless similarity variables	
Re	Reynolds number	
Pr	Prandtl number	
k_p	permeability parameter	
F	radiation parameter	
r	relative difference of temperature	
C	local friction coefficient	
h	heat transfer coefficient	$W \cdot m^{-2} \cdot K^{-1}$
Nu_x	local Nusselt number	
M_w	wall couple stress	N
j	microinertia per unit mass	m^2

Greek letters

ρ	density	$kg \cdot m^{-3}$
μ	dynamic viscosity	$kg \cdot m^{-1} \cdot s^{-1}$
ν	kinematic viscosity	$m^{-2} \cdot s^{-1}$
γ	spin viscosity	
η	transformed similarity variable	
Δ	micropolar parameter	
θ_r	variable viscosity parameter	
γ_1	velocity ratio	
τ_w	wall shear stress	N

perturbation and numerical methods. Hydrodynamic flows of a viscous and incompressible fluid have been studied under different physical conditions with variable fluid properties by Hassanien [4], Seddeek [9] and Aboeldahab and El Gendy [10]. In many practical engineering systems, both the plane surface and the ambient fluid are moving in parallel.

Hence, the aim of the present work is to study the effects of variable viscosity, variable thermal conductivity and radiation on heat transfer from moving surfaces in a steady, incompressible, micropolar fluid through a porous medium. We have reduced the two-dimensional continuity, momentum, angular momentum and energy equations to a system of nonlinear ordinary differential equations which are solved numerically by using ChFD method. The effects of variable viscosity, variable thermal conductivity, radiation, porous medium and micropolar parameters on the flow and heat transfer have been shown in tables and graphically.

2. Problem formulation

Consider a plane surface moving at a constant velocity u_w in parallel or in opposite direction to a free stream of a steady, incompressible, micropolar and electrically conducting fluid of uniform velocity u_∞. Either the surface velocity or the free stream velocity my be zero but not both at the same time. We assume that the fluid properties are isotropic and constant, except for the fluid viscosity, μ, which is assumed to vary as an inverse linear function of temperature, T, in the form [10]

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\big(1 + \delta(T - T_\infty)\big) \quad \text{or} \quad \frac{1}{\mu} = A(T - T_r)$$

where

$$A = \frac{\delta}{\mu_\infty} \quad \text{and} \quad T_r = T_\infty - \frac{1}{\delta}$$

where μ_∞ and T_∞ are the fluid free stream dynamic viscosity and the fluid free-stream temperature. A and T_r are constant and their values depend on the reference state and thermal property of the fluid, i.e., δ. In general, $A > 0$ for fluids such as liquids and $A < 0$ for gases. Also, we assume that, the fluid thermal conductivity, k_f, is assumed to vary as a linear function of temperature in the form [11]

$$k_f = k_\infty\big(1 + a(T - T_\infty)\big)$$

where k_∞ is the fluid free stream thermal conductivity and a is a constant depending on the nature of the fluid. In general, $a > 0$ for fluids such as water and air, while $a < 0$ for fluids such as lubrication oils. This form can be rewritten in the form:

$$k_f = k_\infty(1 + S\theta)$$

where $S = a(T_w - T_\infty)$, is the thermal conductivity parameter and T_w is the value of the plate temperature. The range of variation of S can be taken as follows, for air $0 \leqslant S \leqslant 6$, for water $0 \leqslant S \leqslant 0.12$ and for lubrication oils $-0.1 \leqslant S \leqslant 0$. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x-direction is considered negligible in comparison to the y-direction.

The radiative heat flux q_r is employed according to Rosseland approximation [6], such that

$$q_r = -\frac{4\sigma^*}{3\kappa}\frac{\partial T^4}{\partial y}$$

where σ^* and κ are the Stefan–Boltzmann constant and mean absorption coefficient, respectively.

Under the usual boundary layer approximation, the governing equation for this problem can be written as follows

The equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

The equation of momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left((\mu+K)\frac{\partial u}{\partial y}\right)+\frac{K}{\rho}\frac{\partial N}{\partial y}-\frac{\nu}{k^*}u \tag{2}$$

The equation of angular momentum:

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{j\rho}\frac{\partial^2 N}{\partial y^2}-\frac{K}{j\rho}\left(2N+u\frac{\partial u}{\partial y}\right) \tag{3}$$

The equation of energy:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_P}\frac{\partial}{\partial y}\left(k_f\frac{\partial T}{\partial y}\right)-\frac{1}{\rho c_P}\frac{\partial q_r}{\partial y} \tag{4}$$

subject to the boundary conditions

$$\left.\begin{array}{r} u=\pm u_w,\qquad v=0,\qquad N=0,\\ T=T_w,\quad \text{at } y=0,\\ u=u_\infty,\qquad N=0,\qquad T=T_\infty,\quad \text{as } y\to\infty \end{array}\right\} \tag{5}$$

where, x and y are the coordinate directions, u, v and N are the fluid velocity components in the x and y directions and the component of angular velocity, respectively. K, ρ and υ are vortex viscosity, the fluid density and kinematic viscosity, respectively, j, k^*, γ and c_P are the microinertia per unit mass, the permeability of the porous medium, the spin gradient viscosity and specific heat at constant pressure, respectively.

The boundary condition $u=+u_w$ in Eq. (5) represents the case of a plane surface moving in parallel to the free stream, while $u=-u_w$ represents the case of a surface moving in the opposite direction.

By using the following similarity transformations [2]:

$$\left.\begin{array}{r} \eta=\frac{y}{\sqrt{x}}\sqrt{\frac{u_w+u_\infty}{\nu}},\qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\\ N=\frac{\nu}{x^2}(Re_w+Re_\infty)^{3/2}g(\eta),\\ u=(u_w+u_\infty)f'(\eta),\\ v=-\frac{1}{2}\nu\sqrt{\frac{u_w+u_\infty}{\nu}}x^{-1/2}\big(f(\eta)-\eta f'(\eta)\big) \end{array}\right\} \tag{6}$$

where, the Reynolds numbers are

$$Re_w=\frac{xu_w}{\nu}\quad\text{and}\quad Re_\infty=\frac{xu_\infty}{\nu}$$

Substituting Eq. (6) into Eqs. (1)–(5) produces the following similarity equations and boundary conditions

$$(1+\Delta)f'''-\left(\frac{1}{\theta-\theta_r}\right)\theta' f''+\Delta g'+\frac{1}{2}ff'' -\frac{1}{k_p}f'=0, \tag{7}$$

$$\lambda g''-B\Delta\big(2g+f''\big)+\frac{1}{2}\big(f'g+g'f\big)=0, \tag{8}$$

$$3F\theta''+3F\left(\frac{S}{1+S\theta}\right)\big(\theta'\big)^2+\frac{3F}{2}Pr\, f\theta' +4\big((1+r\theta)^3\theta''+3r(1+r\theta)^2\theta'^2\big)=0 \tag{9}$$

$$\left.\begin{array}{r} f(0)=0,\qquad f'(0)=\pm\gamma_1,\qquad g(0)=0,\\ \theta(0)=1,\\ f'(\infty)=(1-\gamma_1),\qquad g(\infty)=0,\quad\text{and}\\ \theta(\infty)=0 \end{array}\right\} \tag{10}$$

where the primes denote differentiation with respect to η, $\lambda=\frac{\gamma}{\rho j\nu}$ and $B=\frac{x^2}{j(Re_w+Re_\infty)}$. $\Delta=\frac{K}{\rho\nu}$ is the micropolar parameter. $k_p=\frac{k^*}{jB}$ is the dimensionless porous medium parameter. $P_r=\frac{\rho c_P\nu}{k_f}$ is the Prandtl number. $\theta_r=\frac{T_r-T_\infty}{T_w-T_\infty}$ is the variable viscosity parameter. $F=\frac{k_f\kappa}{4\sigma^*T_\infty^3}$ is the radiation parameter, $r=\frac{T_w-T_\infty}{T_\infty}$ is the relative difference between the temperature of the surface and the temperature far away from the surface. $\gamma_1=\frac{u_w}{u_w+u_\infty}=\frac{1}{1+Re_w/Re_\infty}$ is the velocity ratio.

The wall shear stress is related to $|f''(0)|$ by the relation

$$\tau_w=\left((\rho\nu+K)\left|\frac{\partial u}{\partial y}\right|+K|N|\right)_{y=0} =\rho\left(\frac{\nu}{x}\right)^2(1+\Delta)(Re_w+Re_\infty)^{3/2}\big|f''(0)\big| \tag{11}$$

To express the wall shear stress nondimensionally, we define two types of the local friction coefficients as:

$$C_\infty=\frac{2\tau_w}{\rho u_\infty^2}\quad\text{and}\quad C_w\frac{2\tau_w}{\rho u_w^2} \tag{12}$$

A combination of Eqs. (11) and (12) gives

$$C_\infty Re_\infty^{1/2}=2(1+\Delta)(1-\gamma_1)^{-3/2}\big|f''(0)\big|\quad\text{and}$$
$$C_w Re_w^{1/2}=2(1+\Delta)\gamma_1^{-3/2}\big|f''(0)\big|$$

The wall couple stress is related to $g'(0)$ by the relation

$$M_w=\left(\gamma\frac{\partial N}{\partial y}\right)_{y=0}=\gamma\frac{\nu}{x^2}(Re_w+Re_\infty)^2\big|g'(0)\big|$$

The local heat flux may be written by Fourier's law as

$$q_w(x)=-k_f\left(\frac{\partial T}{\partial y}\right)_{y=0}=-k_f(T_w-T_\infty)\sqrt{\frac{u_w+u_\infty}{x\nu}}\theta'(0)$$

The local heat transfer coefficient is given by

$$h(x)=\frac{q_w(x)}{(T_w-T_\infty)}$$

and the local Nusselt number, $Nu_x=\frac{h(x)}{k_f}$, can be obtained from the numerical results by the relation

$$\frac{Nu_x}{Re_\infty^{1/2}}=-\theta'(0)$$

3. The method of solution

Chebyshev polynomials are used widely in numerical computations. Chebyshev polynomials have proven successfully in the numerical solution of various boundary value problems [12,13] and in computational fluid dynamics [14–16]. The present work deals with application of a radically new approach to computation of the boundary layer equations in MHD flows. This approach requires the definition of a grid points and it is applied to satisfy the differential equations and the boundary conditions at these grid points. It can be regarded as a non-uniform finite difference scheme. The derivatives of the function $f(x)$ at a point x_j is linear combination from the values of the function $f(x)$ at the Gauss–Lobatto points $x_k = \cos(\frac{k\pi}{L})$, where $k = 0, 1, 2, \ldots, L$, and j is an integer $0 \leqslant j \leqslant L$ [17–20].

3.1. Chebyshev finite difference method for derivative calculation

The derivatives of the function $f(x)$ at the points x_k are given by [17–20]

$$f^{(n)}(x_k) = \sum_{j=0}^{L} d_{k,j}^{(n)} f(x_j) \quad n = 1, 2, 3$$

where

$$d_{k,j}^{(1)} = \frac{4\gamma_j^*}{L} \sum_{n=0}^{L} \sum_{\substack{l=0 \\ (n+l)\ \text{odd}}}^{n-1} \frac{n\gamma_n^*}{c_l} T_n(x_j) T_l(x_k) \quad k, j = 0, \ldots, L$$

$$d_{k,j}^{(2)} = \frac{2\gamma_j^*}{L} \sum_{n=0}^{L} \sum_{\substack{l=0 \\ (n+l)\ \text{even}}}^{n-2} \frac{1}{c_l} \gamma_n^* n(n^2 - l^2) T_n(x_j) T_l(x_k)$$

$$k, j = 0, \ldots, L$$

$$d_{k,j}^{(3)} = \frac{4\gamma_j^*}{L} \sum_{n=0}^{L} \sum_{\substack{l=0 \\ (n+l)\ \text{even}}}^{n-2} \sum_{\substack{i=0 \\ (i+l)\ \text{odd}}}^{l-1} \frac{1}{c_i c_l} \gamma_n^* n l (n^2 - l^2) \times T_n(x_j) T_i(x_k) \quad k, j = 0, \ldots, L$$

and $\gamma_0^* = \gamma_L^* = \frac{1}{2}$, $\gamma_j^* = 1$ for $j = 1, \ldots, L-1$.

3.2. Chebyshev finite difference approximation for the governing equations

The domain is $0 \leqslant \eta \leqslant \eta_\infty$, where η_∞ is the edge of the boundary layer. Using the algebraic mapping

$$\xi = \frac{2\eta}{\eta_\infty} - 1 \tag{13}$$

the domain $[0, \eta_\infty]$ is mapped into the computational domain $[-1, 1]$ and Eqs. (7)–(10) are transformed into the following equations

$$(1+\Delta) f'''(\xi) - \left(\frac{1}{\theta(\xi) - \theta_r}\right) \theta'(\xi) f''(\xi) + \Delta \left(\frac{\eta_\infty}{2}\right)^2 g'(\xi) + \frac{1}{2}\frac{\eta_\infty}{2} f(\xi) f''(\xi) - \frac{1}{k_p}\left(\frac{\eta_\infty}{2}\right)^2 f'(\xi) = 0 \tag{14}$$

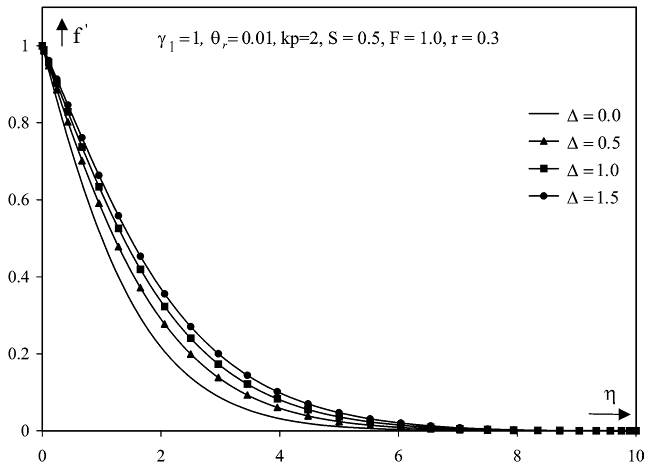

Fig. 1. Effect of micropolar parameter Δ on velocity distribution f' for parallel moving surface.

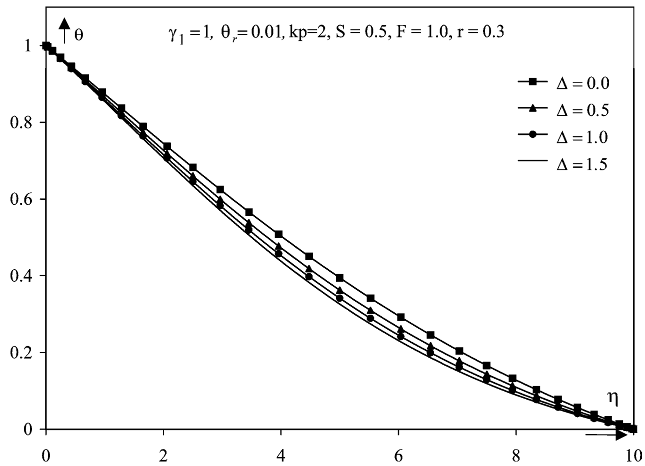

Fig. 2. Effect of micropolar parameter Δ on temperature distribution θ for parallel moving surface.

$$\lambda g''(\xi) - B\Delta\left(2\left(\frac{\eta_\infty}{2}\right)^2 g(\xi) + f''(\xi)\right) + \frac{1}{2}\frac{\eta_\infty}{2}\left(f'(\xi)g(\xi) + g'(\xi)f(\xi)\right) = 0 \tag{15}$$

$$3F\theta''(\xi) + 3F\left(\frac{S}{1 + S\theta(\xi)}\right)\left(\theta'(\xi)\right)^2 + 4\left[\left(1 + r\theta(\xi)\right)^3 \theta''(\xi) + 3r\left(1 + r\theta(\xi)\right)^2\left(\theta'(\xi)\right)^2\right] + \frac{3F}{2}\frac{\eta_\infty}{2} Pr\, f(\xi)\theta'(\xi) = 0 \tag{16}$$

$$\left.\begin{array}{ll} f(-1) = 0, & f'(-1) = \frac{\eta_\infty}{2}(\pm\gamma_1), \\ g(-1) = 0, & \theta(-1) = 1 \\ f'(1) = \frac{\eta_\infty}{2}(1 - \gamma_1), & g(1) = 0, \quad \text{and} \\ & \theta(1) = 0 \end{array}\right\}$$

Thus by applying the ChFD approximation to Eqs. (14)–(16), we obtain the following Chebyshev finite difference equations

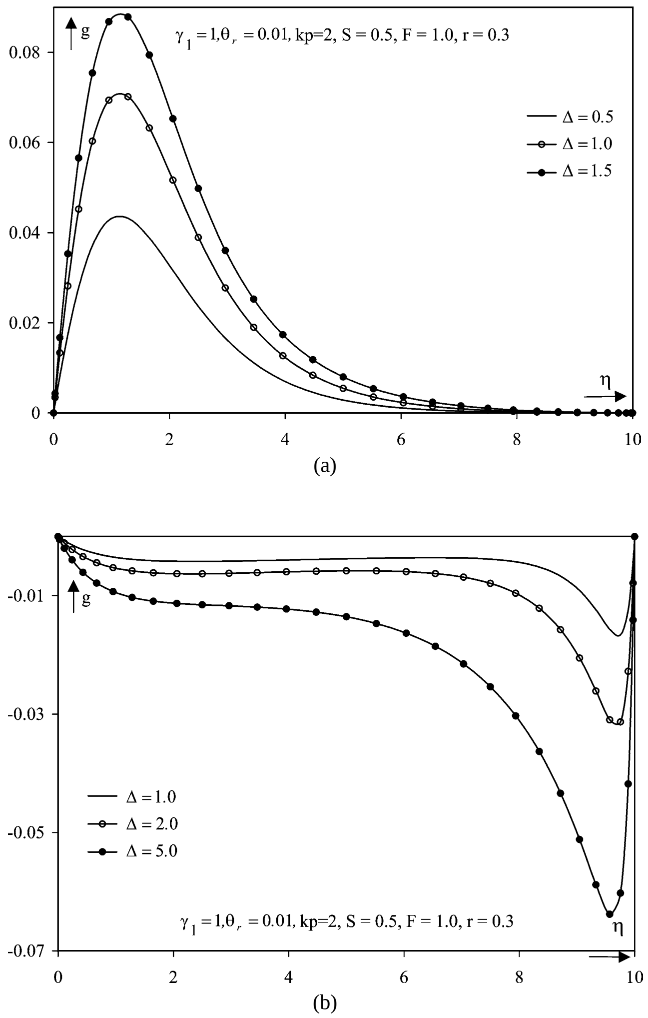

Fig. 3. (a) Effect of micropolar parameter $\varDelta$ on velocity distribution g for parallel moving surface. (b) Effect of micropolar parameter $\varDelta$ on velocity distribution g for reverse moving surface.

$$
\begin{aligned}
&(1+\varDelta)\sum_{j=0}^{L} d_{k,j}^{(3)} f(\xi_j) \\
&\quad - \frac{1}{\theta(\xi_k)-\theta_r}\left(\sum_{j=0}^{L} d_{k,j}^{(1)}\theta(\xi_j)\right)\left(\sum_{j=0}^{L} d_{k,j}^{(2)} f(\xi_j)\right) \\
&\quad + \left(\frac{\eta_\infty}{2}\right)^2 \varDelta \sum_{j=0}^{L} d_{k,j}^{(1)} g(\xi_j) \\
&\quad + \frac{1}{2}\frac{\eta_\infty}{2} f(\xi_k)\sum_{j=0}^{L} d_{k,j}^{(2)} f(\xi_j) \\
&\quad - \frac{1}{k_p}\left(\frac{\eta_\infty}{2}\right)^2 \sum_{j=0}^{L} d_{k,j}^{(1)} f(\xi_j) = 0 \\
&k = 2, 3, \ldots, L-1
\end{aligned}
$$

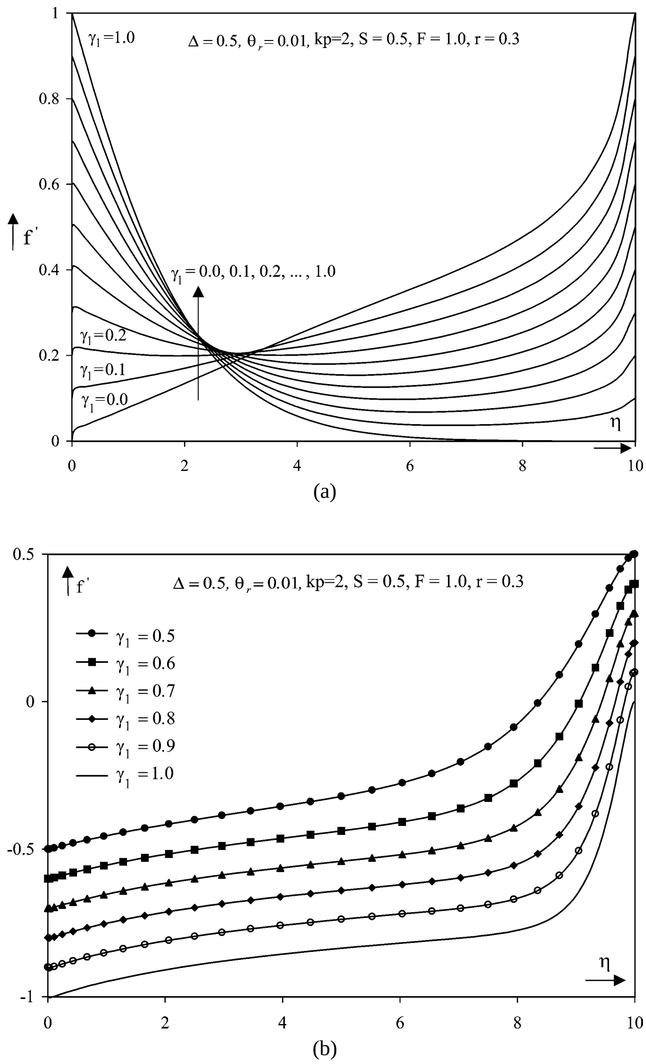

Fig. 4. (a) Effect of the velocity ratio γ_1 on velocity distribution f' for parallel moving surface. (b) Effect of the velocity ratio γ_1 on velocity distribution f' for reverse moving surface.

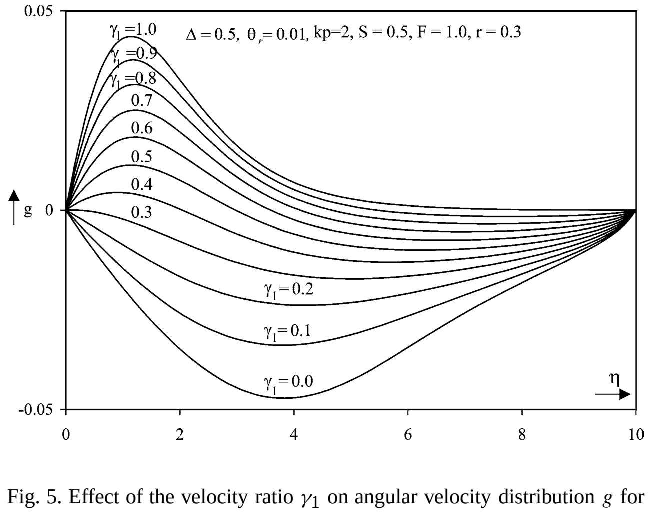

Fig. 5. Effect of the velocity ratio γ_1 on angular velocity distribution g for parallel moving surface.

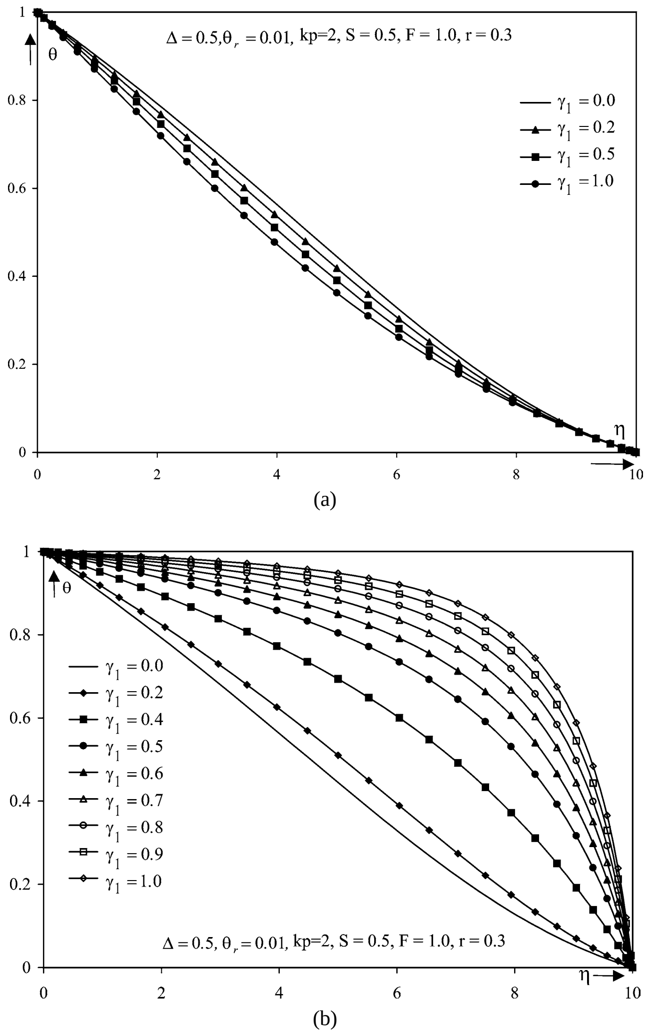

Fig. 6. (a) Effect of the velocity ratio γ_1 on temperature distribution θ for parallel moving surface. (b) Effect of the velocity ratio γ_1 on temperature distribution θ for reverse moving surface.

$$\lambda \sum_{j=0}^{L} d_{k,j}^{(2)} g(\xi_j) - B\Delta\left(2\left(\frac{\eta_\infty}{2}\right)^2 g(\xi_k) + \sum_{j=0}^{L} d_{k,j}^{(2)} f(\xi_j)\right) + \frac{1}{2}\frac{\eta_\infty}{2}\left(g(\xi_k)\sum_{j=0}^{L} d_{k,j}^{(1)} f(\xi_j) + f(\xi_k)\sum_{j=0}^{L} d_{k,j}^{(1)} g(\xi_j)\right) = 0 \quad k = 1, 2, \ldots, L-1$$

$$3F\sum_{j=0}^{L} d_{k,j}^{(2)}\theta(\xi_j) + 3F\left(\frac{S}{1+S\theta(\xi_k)}\right)\left(\sum_{j=0}^{L} d_{k,j}^{(1)}\theta(\xi_j)\right)^2 + \frac{3F}{2}\left(\frac{\eta_\infty}{2}\right) Pr\, f(\xi_k)\sum_{j=0}^{L} d_{k,j}^{(1)}\theta(\xi_j) + 4\left[\big(1+r\theta(\xi_k)\big)^3\sum_{j=0}^{L} d_{k,j}^{(2)}\theta(\xi_j) + 3r\big(1+r\theta(\xi_k)\big)^2 \times \left(\sum_{j=0}^{L} d_{k,j}^{(1)}\theta(\xi_j)\right)^2\right] = 0 \quad k = 1, 2, \ldots, L-1$$

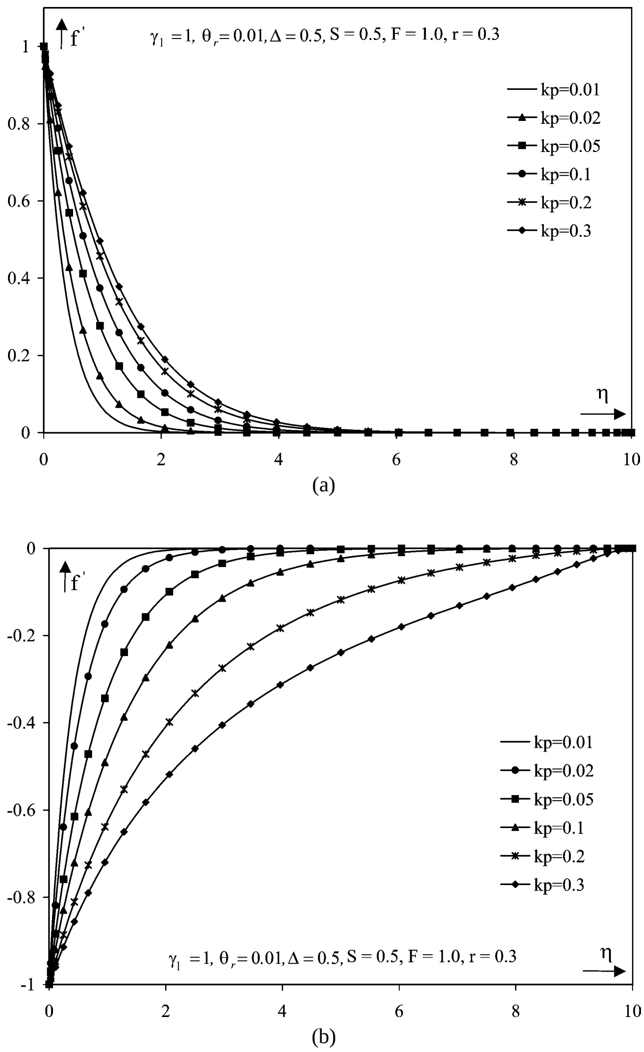

Fig. 7. (a) Effect of porous parameter k_p on velocity distribution f' for parallel moving surface. (b) Effect of porous parameter k_p on velocity distribution f' for reverse moving surface.

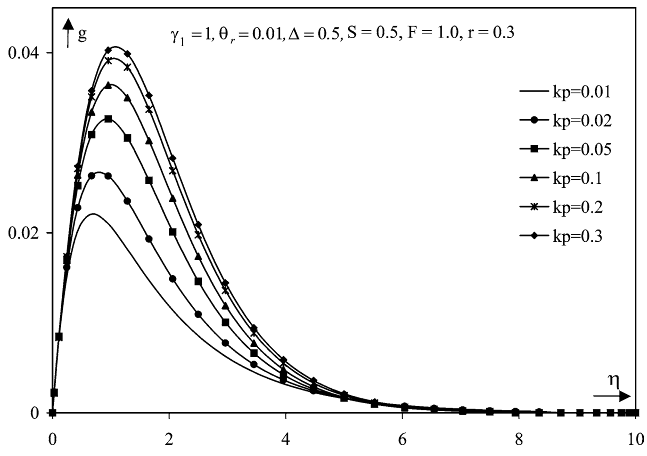

Fig. 8. Effect of porous parameter k_p on velocity distribution g for parallel moving surface.

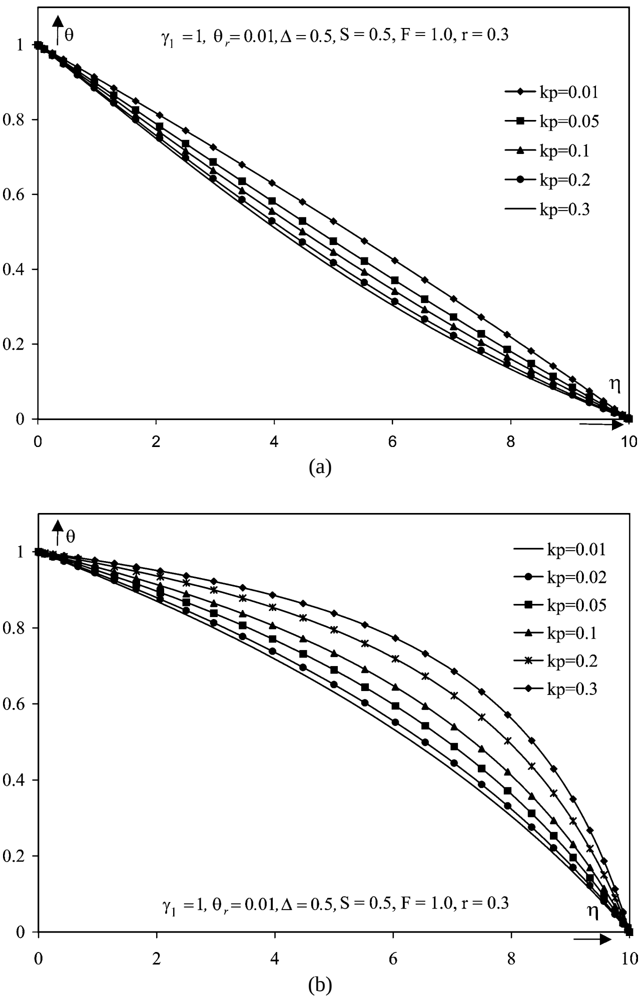

Fig. 9. (a) Effect of porous parameter k_p on temperature distribution θ for parallel moving surface. (b) Effect of porous parameter k_p on temperature distribution θ for reverse moving surface.

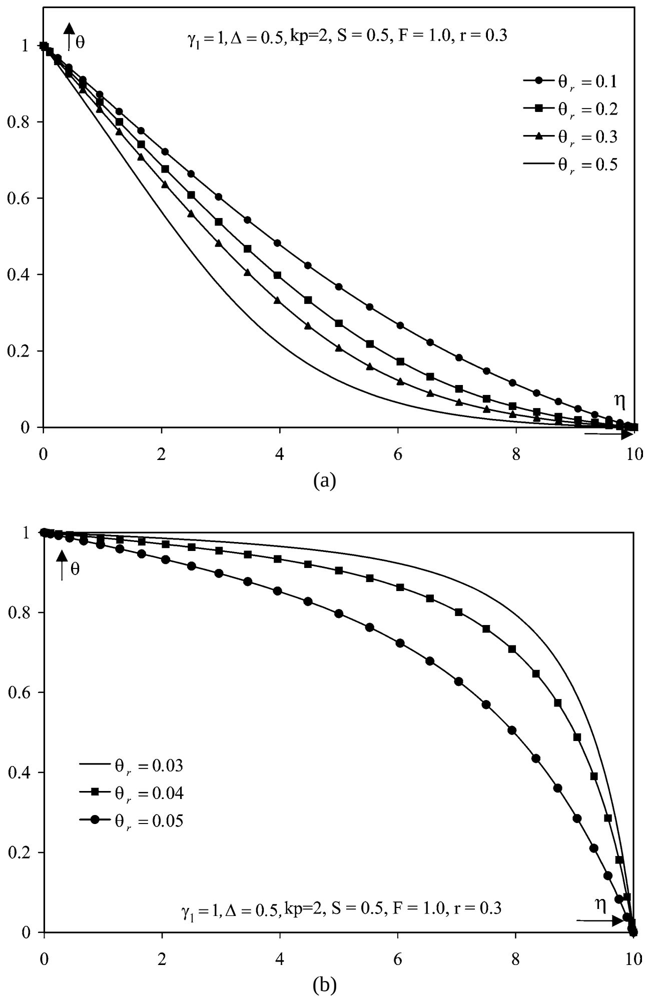

Fig. 10. (a) Effect of viscosity parameter θ_r on temperature distribution θ for parallel moving surface. (b) Effect of viscosity parameter θ_r on temperature distribution θ for reverse moving surface.

The ChFD approximation for the derivative boundary conditions $f'(-1) = \frac{\eta_\infty}{2}(\pm\gamma_1)$, $f'(1) = \frac{\eta_\infty}{2}(1-\gamma_1)$ are formed by

$$\sum_{j=0}^{L} d_{0,j}^{(1)} f(\xi_j) = \frac{\eta_\infty}{2}(\pm\gamma_1)$$

$$\sum_{j=0}^{L} d_{N,j}^{(1)} f(\xi_j) = \frac{\eta_\infty}{2}(1-\gamma_1)$$

The system of nonlinear equations which contains $3L-2$ equations for the unknown $f(\xi_j)$, $i = 1, 2, 3, \ldots, L$ and $g(\xi_i)$, $\theta(\xi_i)$, $i = 1, 2, 3, \ldots, L-1$ is solved by Newton method.

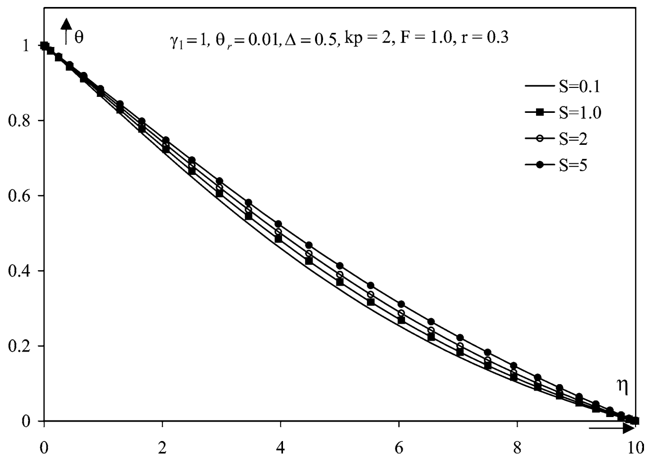

Fig. 11. Effect of variable conductivity parameter S on temperature distribution θ for parallel moving surface.

4. Results and discussion

To study the behavior of the velocity, the angular velocity and the temperature profiles, curves are drawn for various values of the parameters that describe the flow in the case of a plane surface moving in parallel to the free stream and the case of a plane surface moving in the opposite direction to the free stream at $\eta_\infty = 10$, $\lambda = 0.5$, $B = 0.1$ and $Pr = 0.72$.

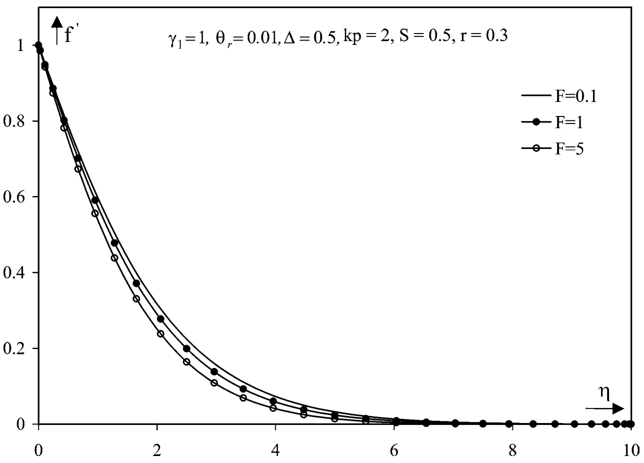

Fig. 12. Effect of the radiation parameter F on velocity distribution f' for parallel moving surface.

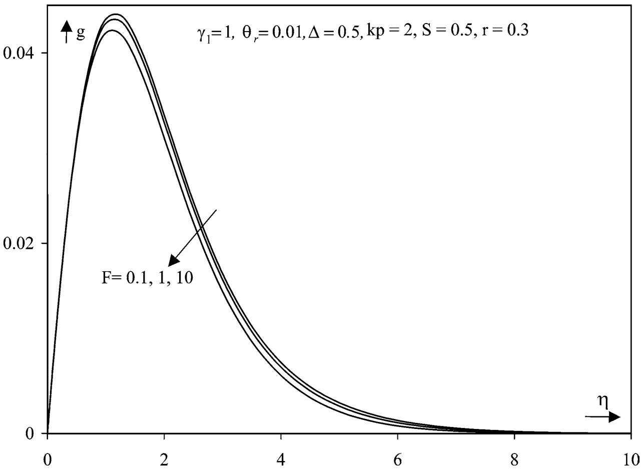

Fig. 13. Effect of the radiation parameter F on velocity distribution g for parallel moving surface.

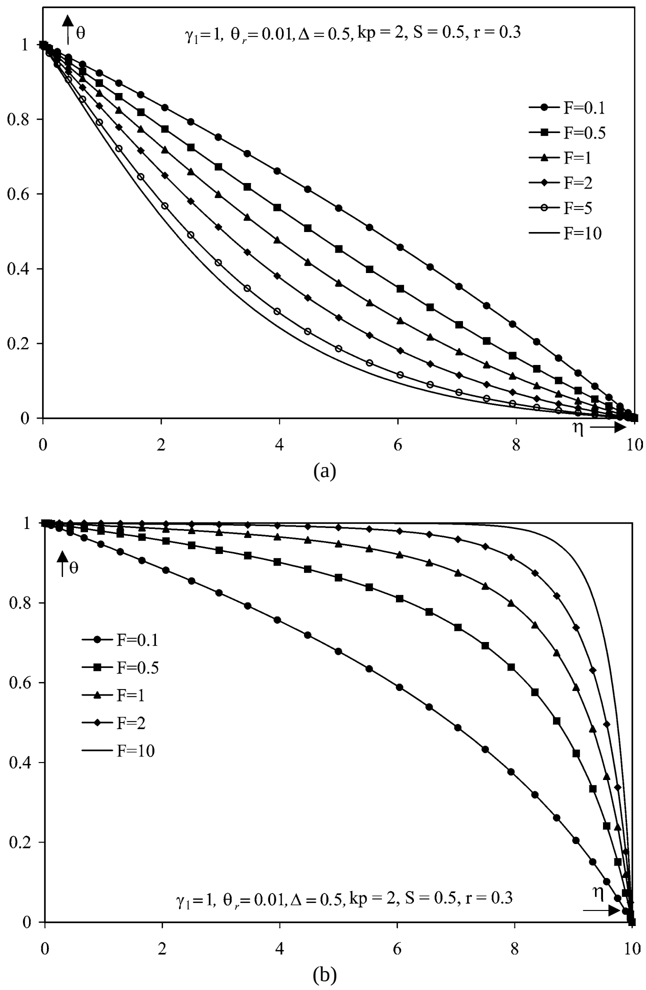

Fig. 14. (a) Effect of the radiation parameter F on temperature distribution θ for parallel moving surface. (b) Effect of the radiation parameter F on temperature distribution θ for reverse moving surface.

In the first case (the case of a plane surface moving in parallel to the free stream):

Figs. 1–3 show the effect of the micropolar parameter Δ on the velocity, the angular velocity, the temperature and distributions, respectively. As shown, the velocity and the angular velocity are increasing with increasing Δ, but the temperature decreases as Δ increases. Figs. 4(a)–6(a) represent the effect of the velocity ratio γ_1 on the velocity, the angular velocity and the temperature distributions, respectively. As shown, the velocity near the boundary layer and the angular velocity are increasing with increasing γ_1, but the velocity far away the boundary layer and the temperature are decreasing with increasing γ_1. Figs. 7(a)–9(a) display results for the velocity, the angular velocity and temperature distribution, respectively. As shown, the velocities are increasing with increasing the dimensionless porous medium parameter k_p and the temperature decreases as k_p increases. The effect of the dimensionless porous medium parameter k_p becomes smaller as k_p increases. Physically, this result can be achieved when the holes of the porous medium are very large so that the resistance of the medium may be neglected. Fig. 10(a) shows the effect of the variable viscosity parameter θ_r on the temperature distribution. As shown, the temperature is decreasing with increasing θ_r. Fig. 11 shows that, the dimensionless temperature distribution increases as the thermal conductivity parameter S increases. Also, Figs. 12(a)–14(a) give the effects of the radiation parameter F on the velocity, the angular velocity and the temperature distributions, respectively. This figures indicate that, all of this quantities decrease as F increases. We notice that, the effect of the radiation parameter F on the temperature distribution is weak. Also, in this case Table 1(a) represents values of $|f''(0)|$, $|g'(0)|$ and $-\theta'(0)$ for various values of the variable viscosity parameter θ_r, the variable conductivity parameter S, the radiation parameter F and the dimensionless porous medium parameter k_p. It is clear that, $|f''(0)|$ and $|g'(0)|$ increase and $-\theta'(0)$ decreases as θ_r increases whereas, $|f''(0)|$ and $|g'(0)|$ decrease and $-\theta'(0)$ increases as k_p increases. $|f''(0)|$, $|g'(0)|$ and $-\theta'(0)$ decrease as S increases whereas, they increase as F increases, i.e., the variable viscosity and the permeability have the opposite effect

Table 1(a)
Represents values of $|f''(0)|$, $|g'(0)|$ and $-\theta'(0)$ in the case of a plane surface moving in parallel to the free stream for various values of the variable viscosity parameter θ_r, the variable conductivity parameter S, the radiation parameter F and the dimensionless porous medium parameter k_p

θ_r	S	F	k_p	$\|f''(0)\|$	$\|g'(0)\|$	$-\theta'(0)$
0.01	0.5	1	2	0.47338229606400317	0.07793950770135853	0.13226778895630242
0.05	0.5	1	2	0.47930686670165185	0.07807090952312165	0.1316960566500151
0.1	0.5	1	2	0.487236721620493	0.07825582862548498	0.13089647145859254
0.01	0.1	1	2	0.4764185916993324	0.07799009077968118	0.13759979138731276
0.01	0.5	1	2	0.47338229606400317	0.07793950770135853	0.13226778895630242
0.01	1	1	2	0.4708391400737787	0.07789431997967056	0.12806556866182123
0.01	2	1	2	0.4677348004078067	0.07783556116907821	0.12321672726038124
0.01	5	1	2	0.46351583137573016	0.0777525635244179	0.11732194154267259
0.01	0.5	0.1	2	0.4396193008993214	0.07730829272371825	0.07786367898585508
0.01	0.5	0.5	2	0.4562858422188947	0.07762283844631164	0.1052183341681238
0.01	0.5	1	2	0.47338229606400317	0.07793950770135853	0.13226778895630242
0.01	0.5	2	2	0.49711582063455806	0.0783538147142093	0.16782884347140253
0.01	0.5	5	2	0.5284939897791537	0.07886219232255452	0.21405332342869604
0.01	0.5	1	0.01	2.665164590790144	0.08962427991128695	0.08851591654858833
0.01	0.5	1	0.05	1.2648431922598353	0.08458101323846938	0.102751620022091
0.01	0.5	1	0.1	0.949489485091824	0.08258760215996477	0.11039572363162209
0.01	0.5	1	0.2	0.7407291863246838	0.08088839795924332	0.11781697390763912
0.01	0.5	1	0.5	0.5778964740399033	0.07923700246426744	0.12572266381200065

Table 1(b)
Represents values of $|f''(0)|$, $|g'(0)|$ and $-\theta'(0)$ in the case of a plane surface moving in the opposite direction to the free stream for various values of the variable viscosity parameter θ_r, the variable conductivity parameter S, the radiation parameter F and the dimensionless porous medium parameter k_p

θ_r	S	F	k_p	$\|f''(0)\|$	$\|g'(0)\|$	$-\theta'(0)$
0.01	0.5	1	2	0.21121222359506647	0.003635970432837052	0.006738528835901418
0.02	0.5	1	2	0.006738528835901418	0.003361376170207813	0.006695365449320134
0.03	0.5	1	2	0.633910478966136	0.002773702328285639	0.0065858905023089605
0.01	0.1	1	2	0.20893912874189483	0.00364372311020349	0.006963680906851406
0.01	0.5	1	2	0.21121222359506647	0.003635970432837052	0.006738528835901418
0.01	1	1	2	0.2156213657814442	0.0036239967257961103	0.006559797294615555
0.01	2	1	2	0.22785528680682546	0.0035941873668730263	0.006347825389555584
0.01	5	1	2	0.27645471931706195	0.003482530994884744	0.0060414936133905365
0.01	0.5	1	2	0.21121222359506647	0.003635970432837052	0.006738528835901418
0.01	0.5	2	2	0.22041882177754815	0.0035503391384754265	0.001098154763181411
0.01	0.5	3	2	0.22978197975697184	0.003519679086656167	0.0002945403569790575
0.01	0.5	4	2	0.23648800179387763	0.003502336662478776	0.00010924830786173678
0.01	0.5	5	2	0.2413897724638082	0.0034905934205445516	0.00004895275053760917
0.01	0.5	1	0.01	2.606127326046759	0.08037696312328567	0.06089099668832979
0.01	0.5	1	0.05	1.1395869394277838	0.057063661658692075	0.0482208131150204
0.01	0.5	1	0.1	0.7751648485172745	0.043730871947144284	0.04001101380355919
0.01	0.5	1	0.2	0.5027361222012814	0.03024781868543334	0.029613312541030924
0.01	0.5	1	0.5	0.2127718730383549	0.015231294262487236	0.015387674057896438

on the shear stress, couple stress and Nusselt number. Also, the variable conductivity and the radiation have the opposite effect on the same quantities.

In the second case (the case of a plane surface moving in the opposite direction to the free stream):
The effect of the micropolar parameter Δ on the angular velocity distribution is investigated in Fig. 2(b). The angular velocity is decreasing with increasing Δ. Figs. 4(b) and 6(b) represent the effect of the velocity ratio γ_1 on the velocity and the temperature distributions. As shown, the velocity is decreasing with increasing γ_1 and the temperature is increasing with increasing γ_1. Figs. 7(b) and 9(b) display results for the velocity and temperature distributions. As shown, the velocity is decreasing with increasing the dimensionless porous medium parameter k_p, whereas the temperature increases as k_p increases. Fig. 10(b) shows the effect of the variable viscosity parameter θ_r on and the temperature distribution. As shown, the temperature is decreasing with increasing θ_r. Finally, Fig. 14(b) gives the effect of the radiation parameter F on the temperature distribution. This figure indicate that, with increasing F the temperature distribution increasing. Also, in this case Table 1(b) represents values of

Table 2(a)
Represents comparison between the results of the author and the results of the Mansour et al. [2] in the case of a plane surface moving in parallel to the free stream. $\eta_\infty = 8$, $\lambda = 0.5$, $B = 0.1$, $Pr = 0.7$, $S = 0.0$, $\theta_r \to \infty$, $k_p \to \infty$, $F \to \infty$

Δ	γ_1	$f''(0)$	$-\theta'(0)$	$g'(0)$
1.5	0.0	(0.18486)0.1859854550	(0.25948)0.2600519545	$(-0.09977) - 0.1002543536$
1.5	0.1	(0.15960)0.1575951510	(0.27700)0.2769988251	$(-0.08617) - 0.0861692688$
1.5	0.2	(0.12417)0.1241740075	(0.29261)0.2926110385	$(-0.06859) - 0.0685907371$
1.5	0.3	(0.08644)0.0864358163	(0.30720)0.3071955606	$(-0.04809) - 0.0480917773$
1.5	0.4	(0.04491)0.0449081558	(0.32096)0.3209600529	$(-0.02511) - 0.0251115247$
1.5	0.5	$(0.0) - 3.97904 \times 10^{-13}$	(0.33405)0.3340523047	$(0.0) - 1.3742 \times 10^{-13}$
1.5	0.6	$(-0.04796) - 0.04796007$	(0.34658)0.3465816316	(0.02696)0.026955094391
1.5	0.7	$(-0.09870) - 0.09870078$	(0.35863)0.3586313691	(0.05551)0.055511148221
1.5	0.8	$(-0.15199) - 0.15199341$	(0.37027)0.3702665906	(0.08546)0.085458951645
1.5	0.9	$(-0.20764) - 0.20764214$	(0.38154)0.3815390850	(0.11661)0.116614447718
1.5	1.0	$(-0.26548) - 0.26547756$	(0.39249)0.3924906689	(0.14881)0.148812815242

Table 2(b)
Represents comparison between the results of the author and the results of the Mansour et al. [2] in the case of a surface moving in the opposite direction to the free stream. $\eta_\infty = 8$, $\lambda = 0.5$, $B = 0.1$, $Pr = 0.7$, $S = 0.0$, $\theta_r \to \infty$, $k_p \to \infty$, $F \to \infty$

Δ	γ_1	$f''(0)$	$-\theta'(0)$	$-g'(0)$
1.5	0.00	(0.18486)0.1859854550	(0.25948)0.2600519545	(0.09977)0.1002543536
1.5	0.05	(0.16840)0.1701452192	(0.23662)0.2376448928	(0.08939)0.0901751251
1.5	0.10	(0.15016)0.1529534191	(0.21097)0.2129178603	(0.07768)0.0789956193
1.5	0.15	(0.12923)0.1340084896	(0.18105)0.1850833210	(0.06421)0.0665458940
5.0	0.00	(0.10651)0.1126611656	(0.23306)0.2374666883	(0.10730)0.1137937182
5.0	0.05	(0.09811)0.1055078665	(0.20916)0.2153036247	(0.09578)0.1034199629
5.0	0.10	(0.08913)0.0981517931	(0.18283)0.1916803245	(0.08415)0.0932065188

$|f''(0)|$, $|g'(0)|$ and $-\theta'(0)$ for various values of the variable viscosity parameter θ_r, the variable conductivity parameter S, the radiation parameter F and the dimensionless porous medium parameter k_p. It is clear that, with increasing θ_r, S and F, $|g'(0)|$ and $-\theta'(0)$ decrease and $|f''(0)|$ increases, whereas with increasing k_p, $|f''(0)|$, $|g'(0)|$ and $-\theta'(0)$ decrease, i.e., the variable viscosity, the variable conductivity and the radiation have the same effect on the shear stress, couple stress and Nusselt number.

Finally, Table 2(a) represents comparison between the results of the author and the results of Mansour et al. [2] in the case of a plane surface moving in parallel to the free stream. Table 2(b) represents comparison between the results of the author and the results of Mansour et al. [2] in the case of a surface moving in the opposite direction to the free stream. In these tables, the given number between brackets refer to the results of Mansour et al. [2] and the given numbers without brackets refer to the present values.

5. Conclusions

This paper studied the effects of variable viscosity, variable thermal conductivity and radiation on heat transfer from moving surfaces in a micropolar fluid through a porous medium. Two cases are considered, one corresponding to a plane surface moving in parallel with the free stream and the other, a surface moving in the opposite direction to the free stream. The fluid viscosity is assumed to vary as an inverse linear function of temperature and the thermal conductivity is assumed to vary as a linear function of temperature. The governing fundamental equations are approximated by a system of nonlinear ordinary differential equations by similarity transformation and are solved numerically by using Chebyshev finite difference method (ChFD), the shear stress, couple stress and Nusselt number as well as the details of velocity and temperature fields are presented for various values of parameters of the problem, e.g., variable viscosity, variable thermal conductivity, radiation, porous, velocity ratio and micropolar parameters. The numerical results indicate that (in the first case) the velocity and the angular velocity increase as the porous parameter increases but they decrease as the radiation parameter increases. The temperature increases also as the variable conductivity parameter increases but it decreases with permeability, variable viscosity and radiation increasing. The variable viscosity and the permeability have the opposite effect on the shear stress, couple stress and Nusselt number. Also, the variable conductivity and the radiation have the opposite effect on the same quantities. Also (in the second case) the velocity decreases as the porous parameter increases. The temperature increases as the porous and the radiation parameters increase but it decreases as the variable viscosity increases. The variable viscosity, the variable conductivity and the radiation have the same effect on the shear stress, couple stress and Nusselt number.

References

[1] H.A.M. El-Arabawy, Effect of suction/injection on the flow of a micropolar fluid past a continuously moving plate in the presence of radiation, Internat. J. Heat Mass Transfer 46 (2003) 1471–1477.

[2] M.A. Mansour, A.A. Mohammadein, S.M.M. El-Kabeir, R.S.R. Gorla, Heat transfer from moving surfaces in a micropolar fluid, Canad. J. Phys. 77 (1999) 463–471.

[3] M.A. Seddeek, Flow of a magneto-micropolar fluid past a continuously moving plate, Phys. Lett. A 306 (2003) 255–257.

[4] I.A. Hassanien, Flow and heat transfer on continous stretching flat surface moving in a parallel free steam with variable properties, Z. Angew. Math. Mech. 79 (11) (1999) 786–792.

[5] M.A. Abd El-Naby, E.M.E. Elbarbary, N.Y. Abdelazem, Finite difference solution of radiation effects on MHD unsteady free-convection flow over vertical plate with variable surface temperature, J. Appl. Math. 2 (2003) 65–86.

[6] A.Y. Ghaly, E.M.E. Elbarbary, Radiation effect on MHD free-convection flow of a gas at a stretching surface with a uniform free stream, J. Appl. Math. 2 (2002) 93–103.

[7] Y.J. Kim, A.G. Fedorov, Transient mixed radiative convection flow of a micropolar fluid past a moving semi-infinite vertical porous plate, Internat. J. Heat Mass Transfer 46 (2003) 1751–1758.

[8] A.C. Eringen, Theory of micropolar fluids, Math. Mech. 16 (1966) 1–18.

[9] M.A. Seddeek, Thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature-dependent viscosity, Canad. J. Phys. 79 (2001) 725–732.

[10] E.M. Aboeldahab, M.S. El Gendy, Radiation effect on MHD free convective flow of a gas past a semi-infinite vertical plate with variable thermophysical properties for high-temperature differences, Canad. J. Phys. 80 (2002) 1609–1619.

[11] J.C. Slattery, Momentum, Energy and Mass Transfer in Continua, McGraw-Hill, New York, 1972.

[12] L. Fox, I.B. Parker, Chebyshev Polynomials in Numerical Analysis, Clarendon Press, Oxford, 1968.

[13] D. Gottlieb, S.A. Orszag, Numerical Analysis of Spectral Methods: Theory and Applications, in: CBMS–NSF Regional Conf. Ser. Appl. Math., vol. 26, SIAM, Philadelphia, PA, 1977.

[14] C. Canuto, M.Y. Hussaini, A. Quarterini, T.A. Zang, Spectral Methods in Fluid Dynamics, Springer, Berlin, 1988.

[15] H. Nasr, I.A. Hassanien, H.M. El-Hawary, Chebyshev solution of laminar boundary layer flow, Internat. J. Comput. Math. 33 (1990) 127–132.

[16] R.G. Voigt, D. Gottlieb, M.Y. Hussaini, Spectral Methods for Partial Differential Equations, SIAM, Philadelphia, PA, 1984.

[17] E.M.E. Elbarbary, M. El-Kady, Chebyshev finite difference approximation for the boundary value problems, Appl. Math. Comput. 139 (2003) 513–523.

[18] E.M.E. Elbarbary, N.S. Elgazery, Chebyshev finite difference method for the effects of radiation and variable viscosity on magneto-micropolar fluid flow through a porous medium, Internat. Comm. Heat Transfer 31 (3) (2004) 409–419.

[19] E.M.E. Elbarbary, Chebyshev finite difference method for the solution of boundary-layer equations, Appl. Math. Comput., in press.

[20] N.T. Eldabe, E.F. Elshehawey, E.M.E. Elbarbary, N.S. Elgazery, Chebyshev finite difference method for MHD flow of a micropolar fluid past a stretching sheet with heat transfer, Appl. Math. Comput., in press.